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ABSTRACT

Summarized are presentations made at a one-day teachers' workshop organized by the Bicultural Socialization Project to discuss the materials to be used in mathematics learning centers in the project classrooms. The first chapter discusses the basic philosophy, whereby pupils are to be encouraged to enjoy the discovery of mathematical relationships through their own activities. Two chapters discuss the mechanics of making assignment cards and of keeping records. Three chapters center on particular themes: pictorial representation (mappings, pictures, charts, set diagrams, tables), number (set description, classification, counting), and linear measurement (length, capacity, weight). A final chapter describes some games which can be used to give practice in number appreciation. Appended is an annotated bibliography on the laboratory approach to the teaching of elementary school mathematics. (MM)

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THE DISCOVERY APPROACH TO MATHEMATICS

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F O R E W A R D

This pamphlet represents the outcome of a workshop in mathematics given February 4, 1971 at the office of the Bicultural Socialization Project, Title VII-M, Wilson School District No. 7, Phoenix, Arizona. A major purpose of the workshop was to present the kinds of materials which might be placed in math learning centers to be established in project classrooms. Such materials would provide for ongoing learning experiences of children in these classrooms.

This workshop was given following a request of the project aides. They expressed the need for knowing more about children's learning in this area and about the kinds of activities and materials which they might use with children.

The seven chapters summarize the presentations made by those conducting the workshop. The games described in the seventh chapter were played by the trainees and evaluated by them. The participants made samples of these games for use in their own classroom.

It is hoped that this pamphlet may serve as a reference for the workshop participants as well as providing information for those not involved in the activities of this day. The materials included in this pamphlet should be considered as a springboard for further development of materials for child learning.

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THE DISCOVERY APPROACH TO MATHEMATICS

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COVER BY DIANA STELLA BRACKER

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Editor

Lois Fair Wilson

CHAPTER ONE

INTRODUCTION

As you talk with others about your early school experiences how often have you said or heard others say, "I was never good at math." or "I hated math. I still do."? Studies reveal this is a common feeling among many adults and youths today. If you are one of those who has no pleasant memories of your arithmetic classes have you considered how you as a teacher might be developing the same attitudes towards mathematics by the children in your classroom?

Recently, British publications have included an analysis of why math was once such a drudge and was such an unpopular subject. Publication of studies by the psychologist, Piaget, on stages of mental development were basic to British educators' concern in developing new ways of helping children develop understanding of math principles. Knowledge of their success has spread throughout England and the United States. This approach which is described in practice in the English Infant Schools could be labeled as the Discovery Approach.

The British Nuffield Mathematics Project has published several books describing different facets of the "new look" in teaching Math. These Nuffield materials have been used for guidance in the development of the materials in this pamphlet. The first published volume of the Nuffield Project includes on the title page a Chinese proverb reflecting the value and

purpose of this approach.

I hear, and I forget
I see, and I remember
I do, and I understand.

In other words, children learn best by "doing" through activity and experience. The knowledge gained by solving genuine problems, and the judgement-making experiences necessary for their solution are a vital part of concept formation.

If a child is given a problem to solve which is related to his environment and this problem is within his framework of reference, it becomes meaningful and he is strongly motivated to seek its solution. Working out the solution becomes exciting and rewarding to him. The child's experience is of major concern to the teacher as she guides the child in his discovery of relationships.

Mathematical activity can be derived from many sources. A wide variety of objects in the classroom, materials in the out-of-school environment and objects in the home setting all provide opportunities for developing mathematical awareness. The child needs to be guided to look at his surroundings with "new eyes". An important role of the teacher is to help the children develop acute powers of observation and an ever-present enthusiasm for finding relationships and problems in the most commonplace situations and places. The teacher herself needs to develop a sensitivity for possibilities that come out of the children's questions and discussions.

Most young children are endowed with a natural curiosity about everything around them. This curiosity is considered by some as a natural drive to learning. Children can be encouraged to investigate their environment and recognize ongoing change. If the child is encouraged to really "see" his surroundings and is helped to develop an ever-increasing sensitivity to it, he will acquire a sense of adventure and derive a great deal of delight in investigating it. As he becomes more and more aware of his surroundings he may be helped to develop an insight into the relationships in his environment and their relationship to the world of mathematics, if <u>mathematics may be defined then as the story of relationships</u>.

Situations that bring about discovery experiences are carefully structured by the teacher. The teaching role is complex. Primarily, it is concerned with the supplying of materials. It must be remembered that withholding materials can be just as important for the stimulus and development of children's thinking as the type of materials that are introduced.

When utilizing this approach to teaching, it is important for the teacher to define the objectives and goals she has in mind as she plans the presentation of any given materials. She always keeps in mind the six stages which take place in any discovery process.

I. Presentation of material under structured circumstances with defined objectives previously established.

II. Providing children sufficient time to manipulate the material and experiment freely with it. Looking, touching, feeling, smelling, tasting accompanied by appropriate discussion takes time.

III. At this stage the mathematical vocabulary relative to the material being presented is introduced. Every attempt to discuss the nature of the material in many different ways is of utmost importance. Through discussion, questions and answers appropriate vocabulary is reinforced.

IV. The next step is the emergence of a problem or the formation of a hypothesis. This could be a question that came out of the discussion or one made by one of the children. At the early stages, it is sometimes desirable for the teacher to pose or ask the question.

V. The testing of the hypothesis can be a thrilling experience for the children. The teacher must make every effort to allow each child to test the hypothesis in his own way. It is rewarding and interesting to see how children will go about their own task of testing the hypothesis.

VI. Communication and sharing findings is the last step.

In going through these stages the most difficult task of the teacher is to refrain from giving the children direct answers. She must constantly remember to encourage each child to make discoveries for himself. As children put into words their findings, they are sealing their discoveries and clarifying them in their own mind. This is vital to the learning process.

CHAPTER TWO

PICTORIAL REPRESENTATION

Often children will begin to write endless stories about everything. Their desire to communicate in writing is a significant cue for the teacher. She should give the child ample opportunity to write about his mathematical experiences. She will guide him to use all of the mathematical vocabulary which he has gained through discussion. The teachers may record on cards or chart the math vocabulary used during discussion. Such records are made available to the child for use in writing his story, creating graphs or charts to illustrate his story.

Until the child finds this to be a rewarding task, it is best not to attempt anything but oral discussion. Communicating discovery experiences can be accomplished in many ways. Here again, the role of the teacher is very important as she attempts to make learning enjoyable.

A story about his discoveries using the language he has learned through discussion is the first and most advisable way to encourage a child to express his experiences. Dictated or self-written stories soon follow. Often these are subjective. Such stories could be kept in the child's individual work folder. The teacher can use these records as a means of assessing the child's depth of understanding of the math concepts being studied.

It is not essential that all math experiences be recorded. There are many ways to record experiences and not all forms are used for the same experience. The teacher can introduce different ways of recording as she goes along with her young researchers letting them realize that tomorrow has many fascinating new projects. The child's interest should determine whether the recording of findings adds or detracts from the enjoyment of the experience.

Communicating these experiences in writing, pictorially or in the form of graphs have as their main objective the reinforcement of mathematical relationships as the child goes from the simple, concrete pictorial representations to the more abstract.

Following are examples of different ways of recording. They may be done individually, in small groups and still others may become class projects when all of the students in the class participate in the investigation.

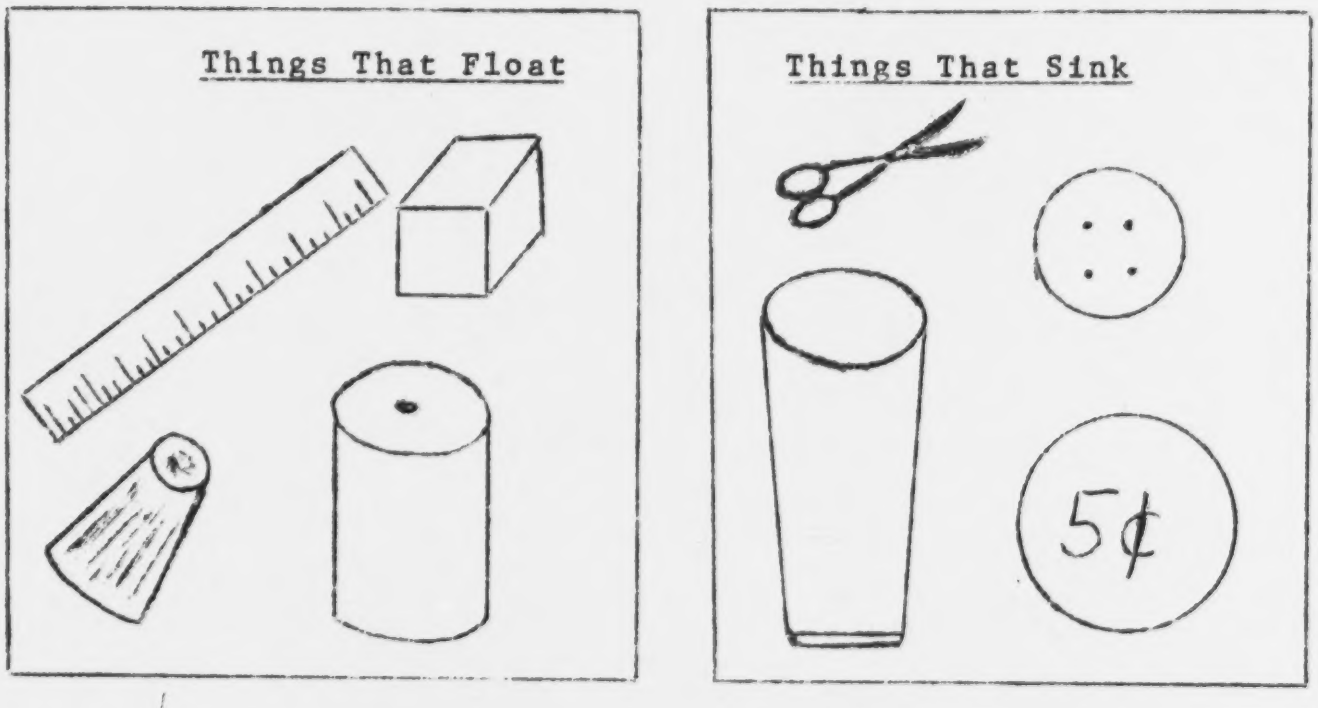
I. MAPPING - a means of showing correspondence on a one-to-one basis.

Example: a junk box can be introduced to the group to promote experiences in classifying according to many different criteria:

color
texture
shape
things that have holes in them and things that don't.
things from the store, the beach, the home, etc.
things that sink and things that float
things I like and things I don't care about
plastic, metal, wood, etc.

Once a given objective for discussion has been established and the six steps mentioned before have been followed, including perhaps, a written story about the ways in which the things from the junk box were classified, mapping of this experience might well follow:

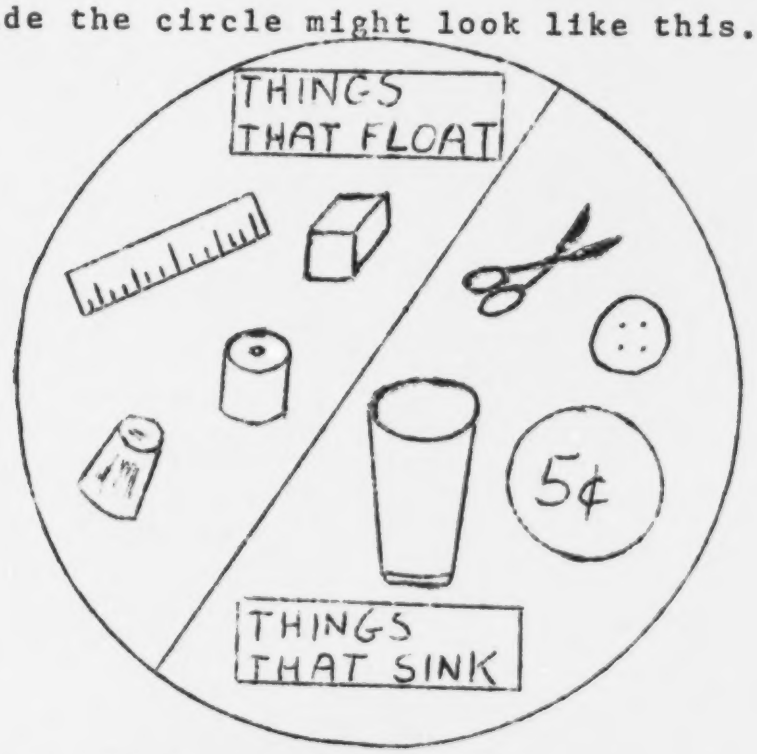
On two cards might be recorded: (1) Things That Float and (2) Things That Sink. The children would then be asked to record this classification. Their record might look like this.



(Above is an example of the drawing the children might make in order to describe their classification. They might cut pictures from magazines instead of drawing them.)

At this time the term collection and set might be explained and discussed. A collection might be described as a set when it is clear that any given one of the objects really does belong to the collection.

II. Another way of recording the preceeding experience is by placing the objects on the table, classifying them and partitioning them with a string. The cards which would be placed inside the circle might look like this.



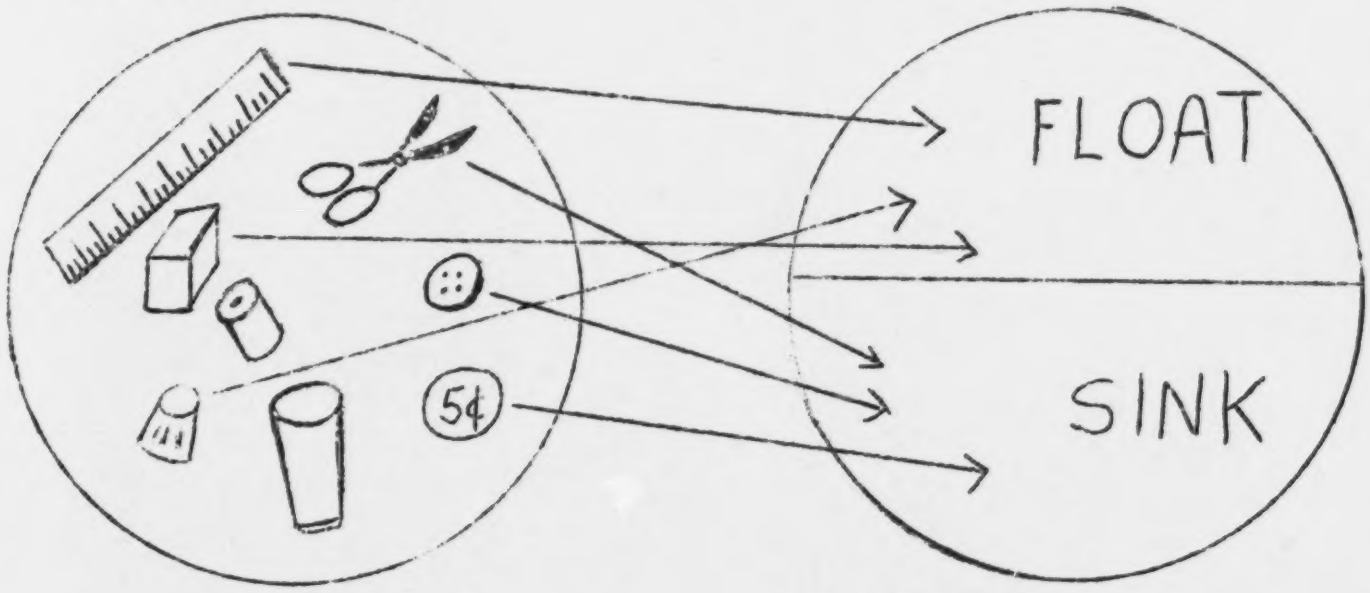
In the previous graph the set is split into two subsets: Things That Float and Things That Sink.

PARTITIONING consists of splitting a set into sub-sets so that each sub-set contains at least one element and each element belongs to just one sub-set.

III. Each item could be labeled on a 5 x 7 card and a block graph might be made on a large piece of tagboard. (See the following page.)

THINGS THAT FLOAT	THINGS THAT SINK
RULER	NICKEL
SPOOL	GLASS
ERASER	BUTTON
PLASTIC CUBE	SCISSORS

IV. Using arrows the same material could be categorized even more pictorially:



As the children discuss their discoveries and the way in which they have recorded them they begin to see the relationships expressed in them. Children might be encouraged to discuss their picture graph. They might be helped to express their findings by indicating the following relationships.

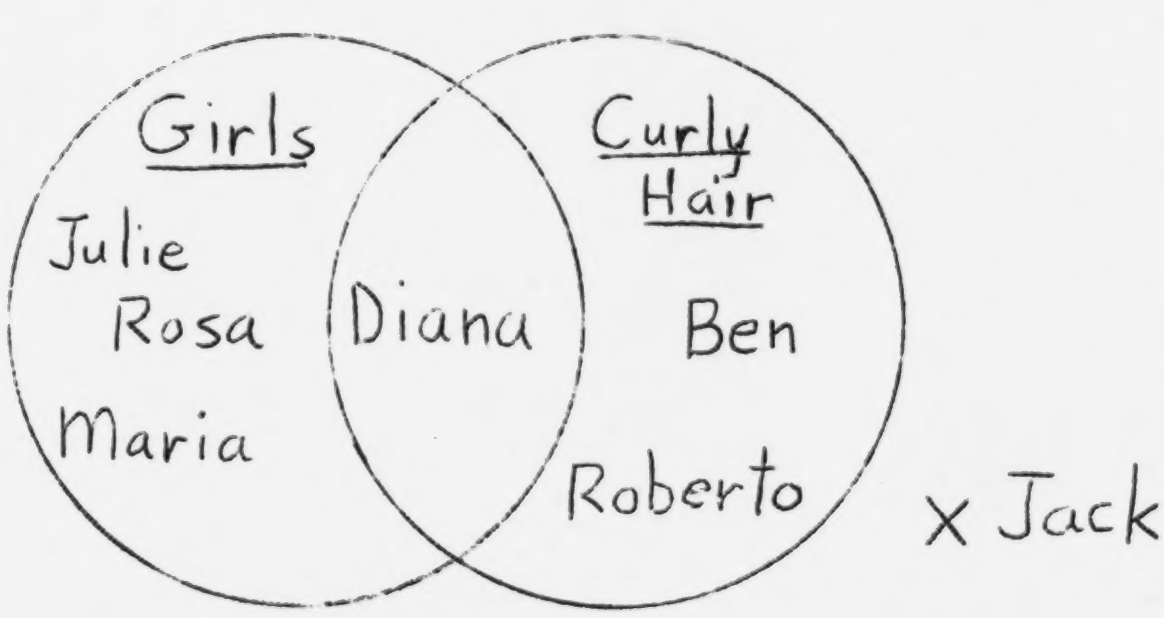
cork - float ruler - float spool - float eraser - float nickel - sink button - sink scissors - sink glass - sink

V. Sets partitioned into more than two sub-sets might be charted in this manner.

METAL	WOOD	PLASTIC

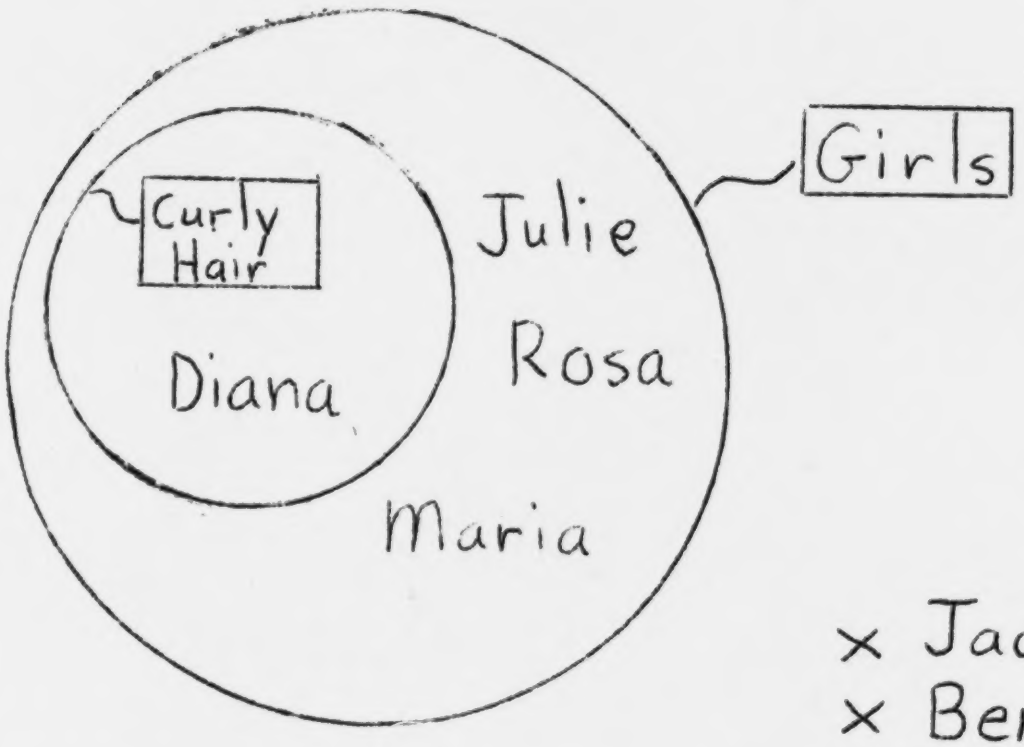
VI. VENN DIAGRAMS - displays of information using different pens to show different categories. This type of representation is useful in introducing the concept of inclusion - that one object can share the characteristics of two subsets. The idea of inclusion practiced here will reinforce the concept that $3 < 7$.

Research can be made in the class amongst the boys and girls to see how many have curly hair. The Venn Diagram can describe their findings.



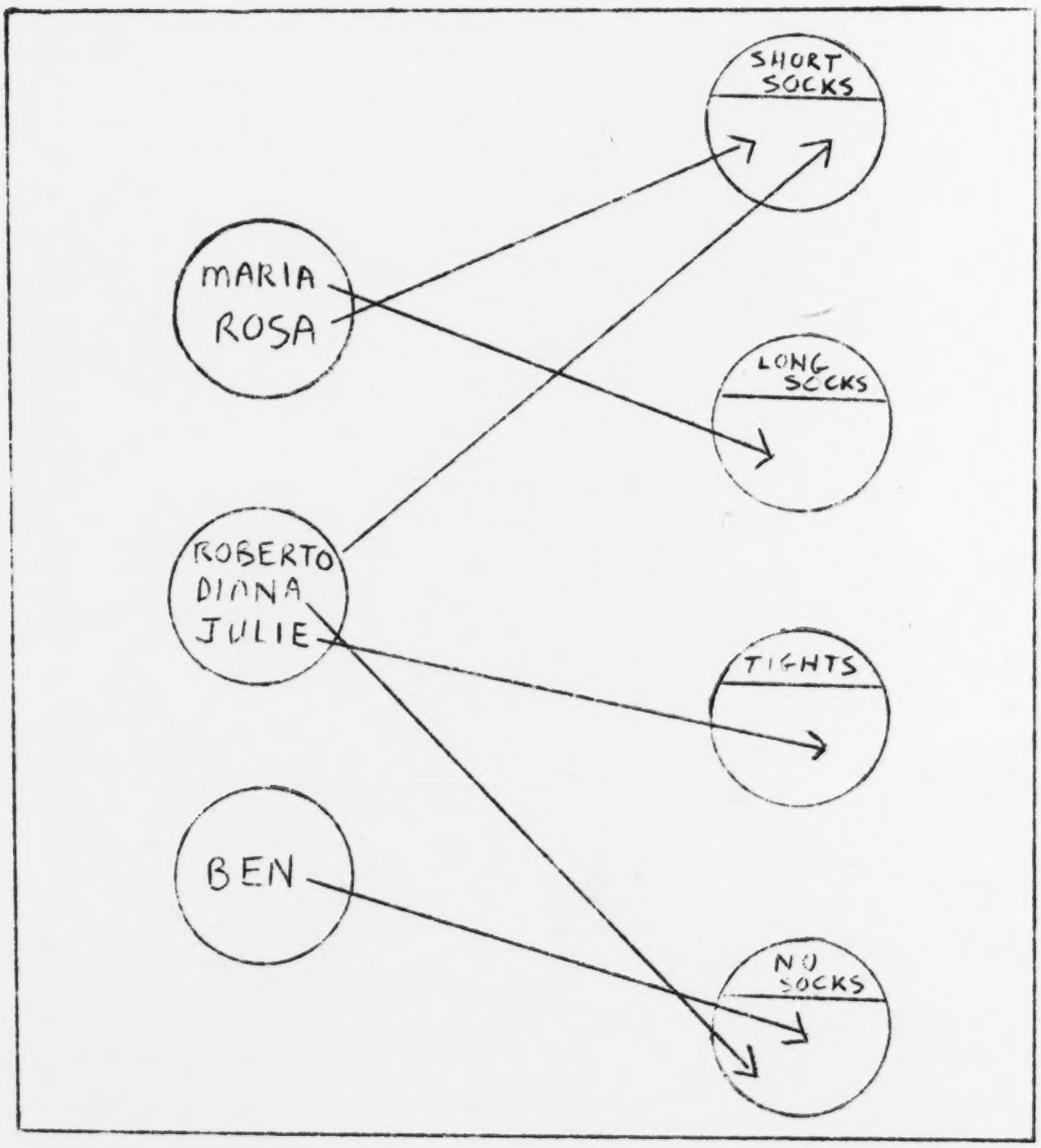
The teacher can ask the children to describe their findings from this diagram. Diana we know is a girl and has curly hair therefore belonging to both sets. Julie, Rosa and Maria are girls but they do not have curly hair. Ben is not a girl but has curly hair and Jack, outside the diagram, is neither a girl nor has he curly hair. (A picture is worth 1,000 words.)

VII. Inclusion can be expressed in other ways:



This diagram is more limited than the previous one. It reveals only which girls have curly hair. We have no information about the boys.

VIII. Sock graphs are fun for the children. The class can make an enjoyable project out of reporting their findings.



IX. As the children develop the ability to graph their findings the teacher can lead them to represent such investigations in a more abstract way.

Name	SHORT SOCKS	LONG SOCKS	Tights	NO Socks
Maria	+			
Rosa		+		
Roberto	+			
Diana				+
Julie			+	
Ben				+

At this time it might be appropriate to discuss with the children the vocabulary used in graphs. Horizontal and vertical axis might be labeled and defined.

Suggested Discovery Projects

Length of socks
ages
shoe size
number of cars seen on the way to school
number of people crossing the street at two or more set times.
color of hair
brothers and sisters
pets in the family
color of eyes
rides, walks or comes to school in bus
birthdays
number of girls having buttons on their dresses
kinds of cars in the family
T.V. programs watched
solids, plaids, stripes in shirt or dress
types of shoes

CHAPTER THREE

ASSIGNMENT CARDS

Math experiences can be recorded in many ways. Children are motivated by the variety of ways in which they can report their discoveries. The use of assignment cards (which may also be labeled job, work, activity or direction cards) has proved to be a very successful way of guiding the young "researchers" to defined objectives of study.

The assignment cards pose questions and suggest problems which involve some form of measurement or mental operations with numbers. It is important to remember that the cards promote active thought which leads to judgement-formation and to the making of decisions. Through these cards the child can be encouraged to make up his own questions as well as to compare his answers with his classmates.

In developing and using assignment cards the following points should be considered by the teacher.

1. Identify specific objectives.
2. Attempt to vary the objectives to include as many mathematical concepts as possible.
3. Have available, in one place, all of the tools necessary to carry out the objective.
4. Pose questions relating to the assignment.
5. Encourage the children to make up their own questions.

6. Encourage preliminary discussion of items included on the card.

7. Encourage the child to compare his results with those of his fellow classmates.

8. Encourage suggestions from groups on further extension or research.

Assignment cards can be made when planning the weekly or monthly class activities, and can include the expected mathematical objectives for that time period. An open-minded attitude towards the cards is advisable, since it is highly possible that new activities and research projects can stem from ongoing work. In fact, the time may come in which the children can suggest their own projects. The teacher is open-minded to this cue. Since each child progresses at his own speed it is important that enough cards are made to cover the same concept so that each child will feel rewarded in his efforts.

The most important value of the assignment card is in promoting problem solving skills for each child and to open the possibilities of all mathematical functions within the discovery experience. When developing assignment cards the teacher develops a range of cards including estimation, measurement, comparison, addition, subtraction, division, and eventually, algebra.

CHAPTER FOUR

RECORD KEEPING

Children enjoy keeping their own records. A book or folder where each child could file his own work has been mentioned previously.

If assignment cards are used, a wall chart might be made where each child could keep a running record of all of the cards he has completed. This is a useful means for a teacher and child to evaluate progress. The chart could be divided in columns of different colors (see diagram below). Color cards might interpret math areas such as shape, size, computation, time problems, money, etc.

	Number NAVY						Money RED						Size GREEN						Shape WHITE						Time YELLOW						computer BLUE		
	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3
P. Blake			✓				✓	✓								✓					✓							✓						
L. Díaz	✓	✓					✓						✓						✓						✓								
C. García			✓				✓	✓								✓								✓			✓						
J. Smith			✓				✓	✓								✓											✓						
B. Vásquez	✓												✓																				
B. Wilson	✓												✓																				

Another wall chart would include names of all the children in the class. Horizontally recorded would be the numbers one to eighteen. Each assignment card would have a specific number. The child could record the date of completion under the number of the designated card. (See diagram below).

NAMES	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
J. ADAMS		2/16			9/12		3/20				1/11							
P. Avila				2/11					1/11				4/20					
B. Baker		2/1																
SM. BROWN						3/1			3/25				4/16			4/24		
J Díaz		4/1		3/5					4/6									
L. Vásquez	2/5			3/1							4/3							

Some teachers like to keep individual records of their student's progress in mathematics. Following is an example of a card that could be used for this purpose.

Assignments completed or Areas investigated	Spring Semester 1971 Name Age Summary of Progress
	Specific difficulties
	Teacher evaluation Teacher's name

CHAPTER FIVE

SETS, FACTS, COUNTING AND TALLYING

ORGANIZING MATERIALS

Many boxes, rocks and pinecones are some of the possible materials available for manipulation. The teacher might arrainge them in some order: size, color, shape, use etc. and asks "Who can describe why I put them in these sets?" After a period of discussion and manipulation one of the children might begin this type of classification. Usually children will be eager to order and arrange materials. The teacher asks questions about the way in which the materials are arranged. The children take turns arranging the items indicating why they choose to organize the materials in a particular way. The discussion can take many directions. Descriptions developing some of the following might develop.

AMOUNT:	more-less	NUMBER:	counting
SIZE:	larger-smaller	OPERATIONS:	adding-subtracting
FEEL:	texture	DESCRIPTION:	color

Before presenting materials the teacher will have in mind one or two concepts she wants to help the children understand. She directs questions to children, picking up on the children's leads as they arrange and talk about the objects. She may record words, phrases or sentences on cards, a chart or the board. She may jot down problems as they are posed and solved. She may help a child write or dictate a story about

what he has done.

Another value of this technique is for identifying concepts children know and further experiences needed to develop concepts. Their math vocabulary can be determined and then extended.

USE OF MATERIALS

A collection of boxes is good for this type of activity. Questions such as the following might be asked.

How many colors on each box?

How many words on a box?

How many sides does a box have?

How many boxes open with a flap? (from the top, from sides)

How long (wide, deep) are the boxes?

Measure boxes with tape measure.

Sort by color, shape, size.

How could these boxes be organized?

Why did John organize his boxes in this way?

Rocks can be used and sorted according to texture, size, color. Objects might be classified according to things which come from trees, the ground, the kitchen. Objects might be sorted according to the kind of material and use. Games can be centered around a specific question.

Why are these things sorted this way?

How are these things alike?

How are these things different?

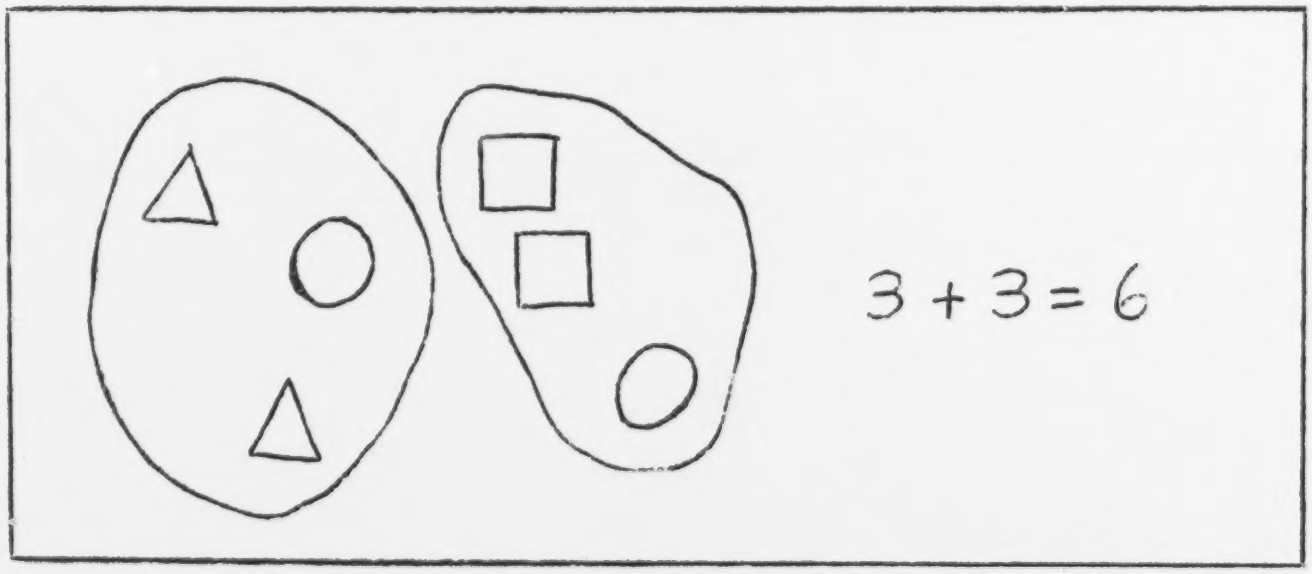
How many scales do you estimate to be on this pinecone?

How many rocks are in this set? Estimate first.

Can you divide the set into three subsets?
(string may be used to partition a subset)

WORKING WITH SETS

Each child is given the choice of several shapes cut from gummed paper. One child sticks his shapes on a large sheet of paper any way he likes. Another adds his and a third circles 2 (or more)sets with a crayon. The group (or a fourth child) writes the problem.



Written assignment cards or charts can be made, giving instructions at a math center for any of these activities, after the children have had ample time to manipulate the materials,

make discoveries of their own both alone and with the guidance of an adult. A card at a center with a collection of bottles might read:

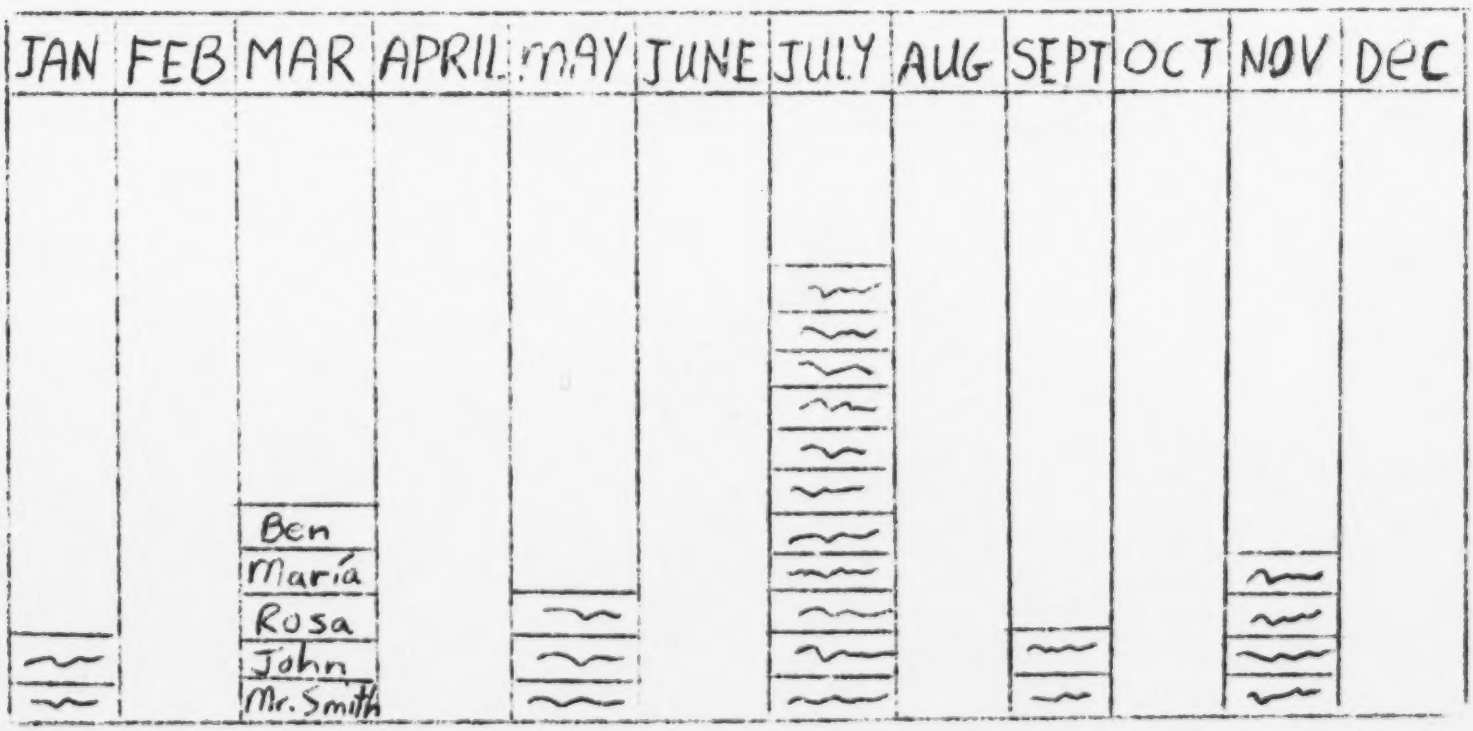
> Make 2 sets using the bottles.
> Write an equation about the 2 sets.

USING A CALENDAR

A yearly calendar may be used to stimulate comparison, observation, adding, subtracting, dividing, the "0" concept, seasons, numberical order.

GRAPHING

On small pieces of construction paper, each child might write his name and paste the paper in the column which represents the month in which he was born.



From this, observations and comparisons may be drawn.

How many children had birthdays in Jan.? March?

Which month has the most birthdays? How many?

Which month has no birthdays?

How many months are in a year?

These months have no birthdays_____

These months have 1 birthday_____

These months have 2 birthdays_____

How many children were born in winter? spring? summer? fall?

Concepts such as semi-annual, annual, seasons, half year, mid-year can come from this work.

USING NUMBERS

Lessons could be geared around one number and all the things you can do with it. How does it look and feel? Problems can be created with it. Words may be developed about it. How might we write it? (Roman Numeral). Can we find it around the room? Can we play a game using the number?

Tallying as a device can be introduced in any counting activity or game scoring. The counting by five after the tallys are taken is good practice.

CHAPTER SIX

LINEAR MEASUREMENT

The child needs to start with arbitrary things for measuring. These should be objects familiar to his own world, i.e. string, yarn, match boxes, sticks, lengths of paper, knitting needles, wood, etc. An example of a problem in this form of measurement would be: How many match boxes long is this table? Estimate first. (We discussed the need to estimate first.) By guessing, the child is usually more anxious to find the correct (or nearest correct) measurement.

Such units as: digit, palm, span, foot, cubit, yard, fathom, etc. might be used as the next form of measurement. It is easier for children to relate body parts to measurement before moving on to the standard units of measure. Things that can be visually perceived and related to oneself are easier for children to understand. An example of this kind of problem would be:

> Estimate first. Measure the length of your desk using:
>
> a. cubits
>
> b. palms
>
> c. spans
>
> d. digits

By encouraging students to compare their measurements for these problems children can be helped to recognize the need for a standard unit of measurement.

Items that could be put in the math center for teaching the concept of linear measurement include match boxes, string, knitting needles, ribbon, wood, yarn, book lengths, lengths of paper, charts identifying limb measurements, i.e. cubits, spans, foot, yard, palm, digit and fathom. When the standard unit of measurement is introduced, items such as the following could be added: rulers, yardsticks, tape measures and all types of objects for measurement in standard units.

MATH ASSIGNMENT CARDS

The math assignment card or work-problem card was introduced in the workshop. Each participant took a card and answered some of the questions on it. Participants discussed and compared the answers.

These cards might be used for independent or small group activities. The cards could be stored in a file box and kept in the math center.

LIQUID MEASUREMENT

Introducing liquid measurement is much like initiating linear measurement. Working from arbitrary units to the standard unit of measure would be meaningful. Following is an

example of a math assignment card using arbitrary units.

> Estimate first.
>
> How many cupfuls fill one jam jar?
>
> How many jam jars fill one jug?
>
> Make a table.

After spending much time on arbitrary units of liquid measurement, children can be helped to move to standard unit of measurement, i.e. pints, quarts, gallons, etc. Following is an example of a problem using standard units.

> Using water:
>
> How many pints can I pour into one gallon?
>
> Estimate first.
>
> How many pints can I pour into one quart?
>
> Estimate first.

Items that could be included in the math center for teaching liquid measurement include some of the following: sand, water, beans, various sizes of bottles, cans, jars, buckets, jugs, cups, pints, 1/2 pints, quarts, gallons and measuring spoons.

WEIGHT

It is important to recognize the need for developing understanding of arbitrary weights before going on to the

standard units. Following is an example of this type of problem.

> Use a balance scale.
>
> How many beans weigh about the same as 6 rocks?
>
> Before weighing, estimate.

One could try this problem with such objects as: nails, sand, seeds, water, feathers and iron.

Teaching for understanding of the comparative concepts of heavy, heavier, heaviest; light, lighter, lightest; long, longer, longest; wide, wider, widest, etc. is developed through the use of varied materials. The following objects could be used: graduated seashells, graduated boxes, graduated keys, graduated screws, etc.

When the students are familiar with arbitrary weights and concepts of size, it is time to move into the standard units, i.e. ounce, pound, ton, etc. Some objects that could be placed in the math center for the teaching of weight are: all types of manipulative objects that can be weighed or compared, balance scales, food scales, postal scales, fisherman's scales.

Time and money concepts can also be extended in this manner. Introducing clocks, coins and paper money provide a concrete base for the development of such understanding.

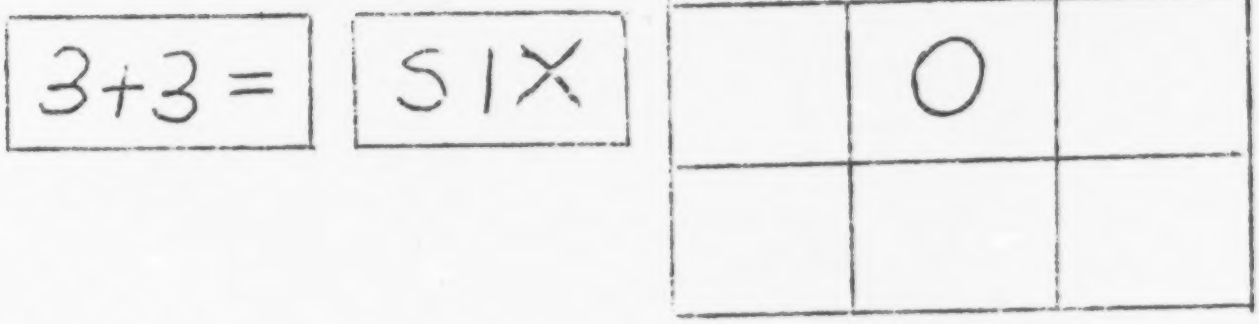
CHAPTER SEVEN

GAMES

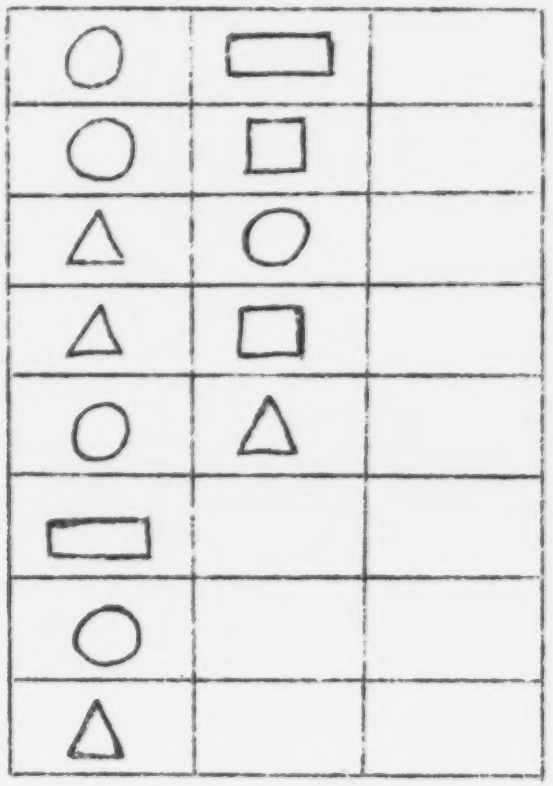
Games which give the children practice in adding and subtracting are useful. Such games can be an independent activity, once the children are thoroughly familiar with the games.

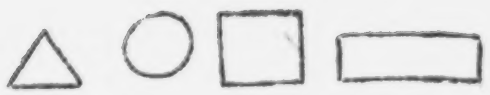
The teacher and aide might play the games before introducing them to the children attempting to identify possible problems children might encounter as they play. For example, if a leader holds up a card with a problem and the children are to cover the answer on their cards, the answer should be on the back of the leader's card. This avoids many trips to the teacher and disputes among the group members.

I. Bingo-type games can be made by the teacher as well as the children. Where possible, more than one set of rules for play should be devised, one set of cards for problems, another for the number word. Answer cards remain the same.

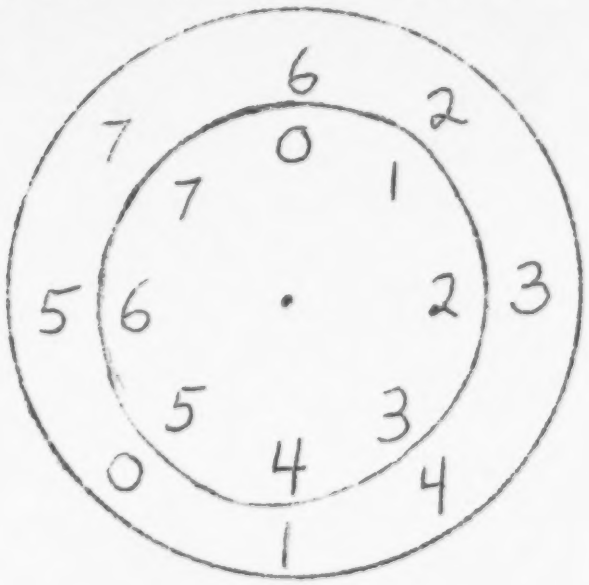


II. <u>Dice offer many opportunities for number, drill and counting.</u> At a throw of the dice a marker is moved the number spaces of the throw. Markers can be various geometric shapes. When one lands on a space with his shape, he gets a pre-determined bonus move or a penalty. The children can devise rules for these games and write them up for use.





III. <u>Wheels</u> can be made which give a selection of all combinations within the child's abilities.

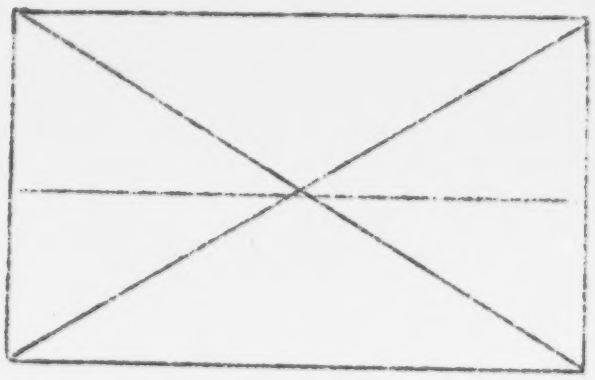


A moveable sign might indicate addition or subtractions.

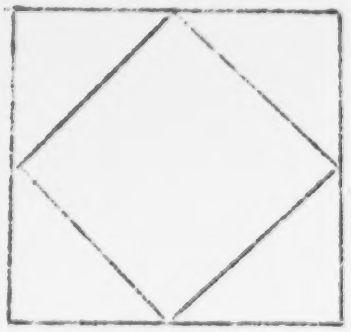
IV. Geometric shapes are a fascinating field of discovery for children when they can manipulate them and see how they combine. Folders can be made with shapes which combine to form another shape. The design to be made can be on the cover of the folder for the child to follow and/or written instructions can be provided. When using these folders, the teacher can talk about and label the different shapes used as well as encourage the child to label them. Here are 8 ideas.

— — — —

Can you make a ▭ with 4 small △'s with 2 large △'s?

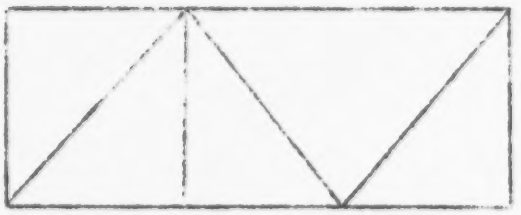


With 4 △'s can you make an inner square?

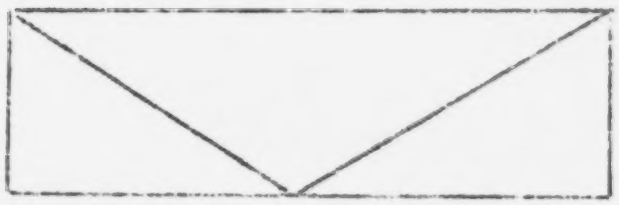


With 1 large △ with 3 small △'s can you make this?

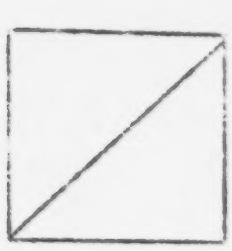
With 4 small △'s with 1 large △ can you make a ▭ ?



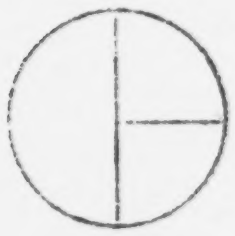
With 2 small △ 's
with 1 large △
can you make a ▭ ?



With 2 △ 's
can you make a □ ?

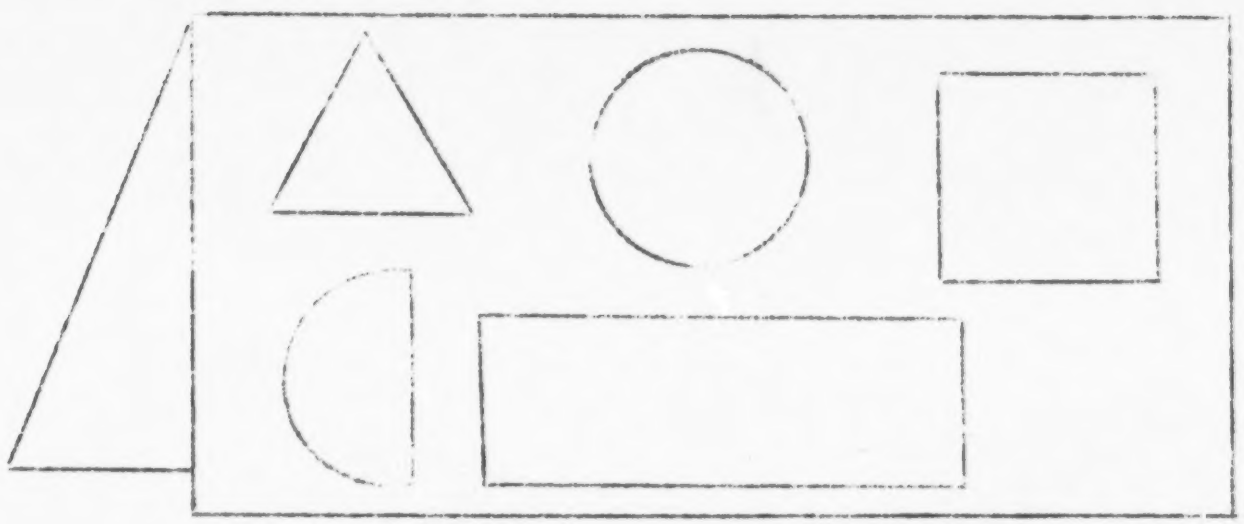


With 1/2 circle
and 2 1/4 circles
do you have a ○ ?

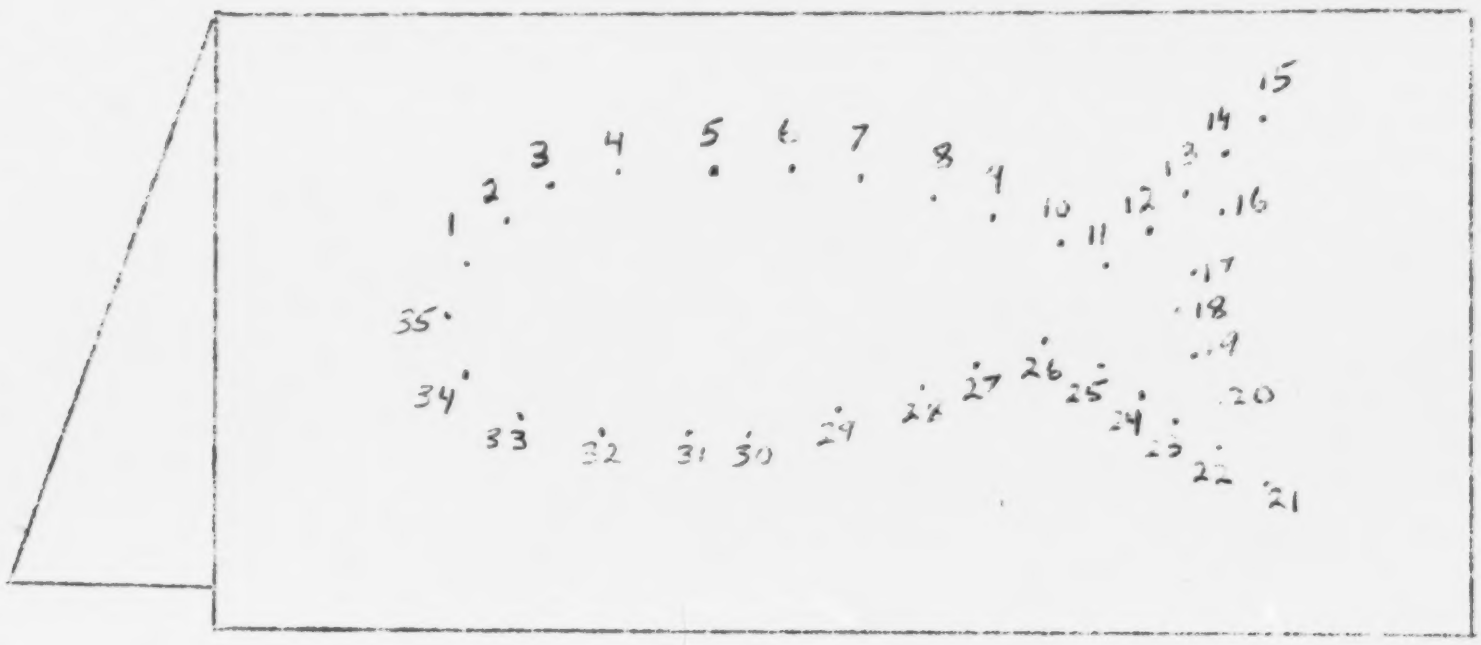


With 2 △ 's
can you make a large △ ?

V. Geometric shapes can also be used here to provide practice in forming them.



VI. Number sequence can be practiced by doing dot-to-dot drawings. A sheet of acetate can be attached to poster or tag board and a picture layed under it. The teacher might find suitable pictures in color books to make these. Caution use only plastic marking crayons for easy removal, permanent felt pens are permanent.

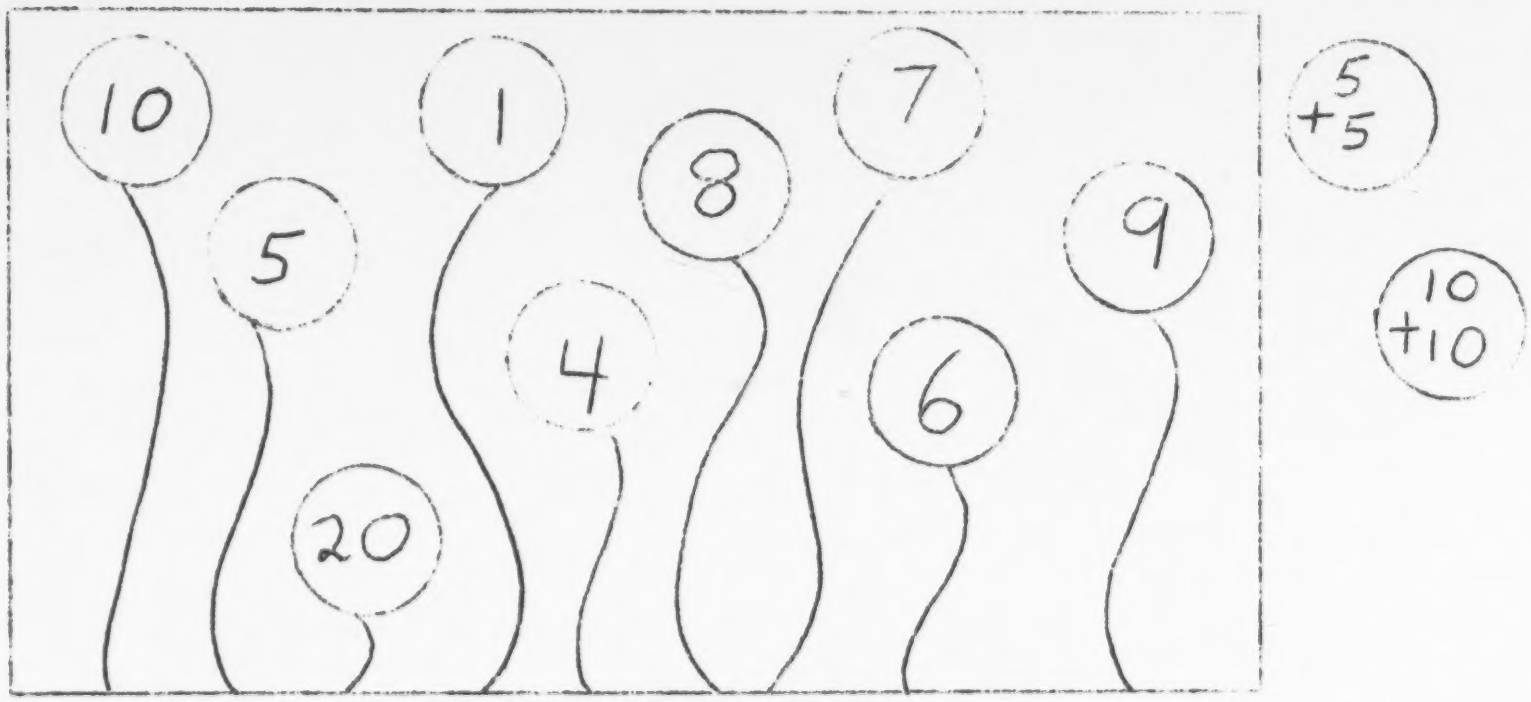


VII. Felt numbers on cards are another teacher-made material, useful in helping children recognize and form the numbers. Some teachers use sandpaper in place of felt.

VIII. A game called "Niner" can be adapted to many levels. (Adaptation of game developed by Addison-Wesley, 1970) Many cards from 1-10 are provided, mixed and distributed among players,

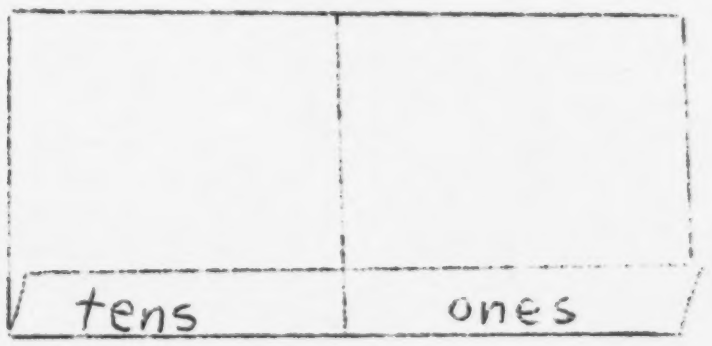
each child puts out a combination forming 9 (or) one card is put in the middle by a child, another adds his and any combination forming 9 gets all the cards. It could be used for subtraction also. This is a flexible game and should be played by adults first to determine how it could best be used in the classroom.

IX. Balloon Game Prepare a 12" x 18" card with many balloons having sums on them. The children receive problems on balloon shaped cards. Each child selects a problem from his cards and puts it on the balloon having the answer. He says the problem and the answer as he does this. This could be adapted to any theme, flower, airplanes, cars, etc.

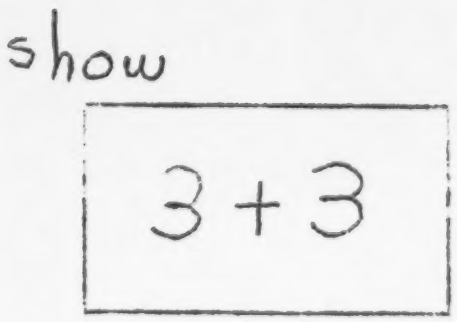


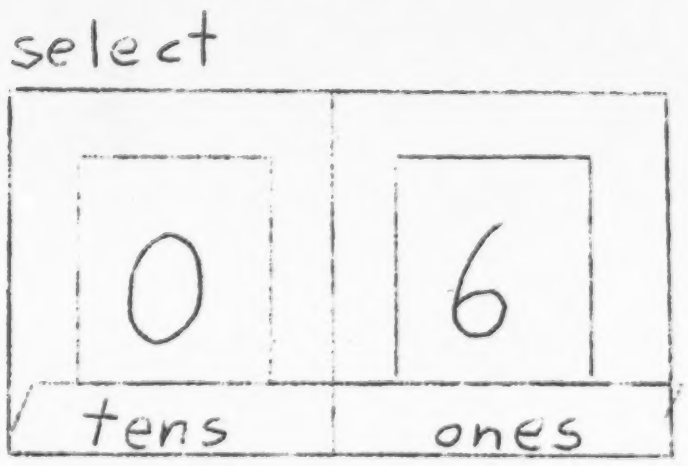
X. <u>Place value and mental computing</u> are practiced with the following game.

Provide a 2 place pocket card for each player.



In the pockets are cards 0 through 9 and arranged in ascending order from front to back. Children hold the pockets in front of them, facing the leader and when a problem is shown, they select the proper answer from the pockets.





Game developed by Cecelia Gonzales

The basis for this approach to mathematics may be summarized in the following manner.

1.. Establish a permanent math center.

2. Make it interesting and practical.

3. Promote interest.

4. Develop understanding.

5. Encourage discovery and the development of a variety of ways of solving a problem.

6. Mathematics is treated as a whole.

7. Foster discussion. Encourage children to talk and learn from each other.

8. Discover mathematical relationships in other areas.

9. Use real situations from the surrounding environment.

10. Mathematics can be a creative and fascinating subject.

B I B L I O G R A P H Y

An Annotated Bibliography on the Laboratory Approach To The Teaching of Elementary School Mathematics.

Prepared by Dr. Edwina Deans to supplement her article in Today's Education: NEA Journal. February, 1971.

(1) Baker, Gail L., and Goldberg, Isadore. "The Individualized Learning System." Educational Leadership 27 (May 1970) 775-780. Defines an individualized learning system, describes some of the demonstrations of individualized education throughout the country and gives suggestions for implementing an individualized instruction program. Eight articles are devoted to the central theme of the issue, Projects, Packages, Programs.

(2) Biggs, Edith E., and MacLean, James R. Freedom to Learn. An Active Learning Approach to Mathematics. Reading, Mass: Add Addison-Wesley Publishing Co., Inc. 1969. The authors define "active learning, discovery methods, and laboratory approach" to mean "an approach to learning that presents a wide variety of opportunities, an approach that encourages them (children) to find the answers, an approach that fosters the use of physical materials, an approach that gives experiences designed to help them analyze and abstract, and an approach that provides a chance to develop their individual potential." Based on experimental work in British schools, this book is an excellent resource for school staffs that are planning to intitiate a laboratory approach both from the standpoint of theoretical foundations and practical considerations.

(3) Cochran, Beryl S., Barson, Alan, and Davis, Robert B. "Child-Created Mathematics." The Arithmetic Teacher 17 (March 1970): 211-215. Relates a third grader's discovery of a new algorithm for substracting involving negative numbers and a second grader's determination to continue graphing open sentences until she found one that would give her a line that "stood up straight" (vertical). Implications for teaching for discovery are drawn from these two illustrations.

(4) Copeland, Richard W. How Children Learn Mathematics--Teaching Implications of Piaget's Research. New York: The Macmillan Co. 1970. Chapter 14, "The Mathematics Laboratory--An Individualized Approach to Learning," offers helpful suggestions for starting and operating a methematics laboratory. Sections in the chapter deal with materials and their uses, teacher and pupil responsibilities for the care of materials, learning basic operations, student assignemnt cards and reports kept by students.

(5) Davidson, Patricia S. "An Annotated Bibliography of Suggested Manipulative Devices." The Arithmetic Teacher 15 (October 1968): 509-524. Suggested devices for mathematics instruction are organized under fifteen general categories: Blocks, Calculators/Computers, Cards, Construction, Drawing Tools, Geoboards, Measuring Devices, Miscellaneous Items, Models, Numerical Games, Puzzles, Shapes and Tiles, Strategy Games, Student Instructional Materials, and Teacher Resource Materials. Each item is annotated and information on the supplier is provided.

(6) Davidson, Patricia S., and Fair, Arlene W. "A Mathematics Laboratory--From Dream to Reality." The Arithmetic Teacher 17 (February 1970): 105-110. Gives details on the authors' experiences in preparing the mathematics facility, equipping the laboratory, and on different ways of working with children and teachers in the operation of the laboratory. Several other articles in this issue offer suggestions for units and activities for use in the laboratory.

(7) Gibb, Glenadine E. "Through the Years: Individualizing Instruction in Mathematics." The Arithmetic Teacher 17 (May 1970): 396-401. Provides background information on the major attempts to individualize mathematics instruction in the past, describes recent project efforts, and identifies questions to be answered if we are to realize the goal of individualized instruction for all children in the future.

(8) Lavatelli, Celia Stendler. "Contrasting Views of Early Childhood Education." Childhood Education (February 1970) 239-246. Describes types of early childhood curricula including the British Infant Schools which derive their theoretical basis from the research of Piaget.

(9) Leeb-Lundberg, Kristina. "Kindergarten Mathematics Laboratory--Nineteenth-Century Fashion." The Arithmetic Teacher 17 (May 1970): 372-386. Describes Froebel's mathematics program--the Gifts and Occupations of the Froebelian era. Today's primary teachers will see possibilities for adapting some of the activities to their own materials and content.

(10) May, Lola J. "Learning Laboratories in Elementary Schools in Winnetka." The Arithmetic Teacher 15 (October 1968): 501-503. States the purposes and describes the functioning of the Winnetka mathematics laboratories. Children's written reactions to their laboratory experiences are included.

(11) National Association of Elementary School Principals. Teaching Mathematics in the Elementary School--What's Needed?

What's Happening? National Association of Elementary School Principals National Education Association and the National Council of Teachers of Mathematics. Washington, D.C.: 1970. The chapter, "Using Manipulative Materials in Mathematics Education," (65-80) by Sandra J. Margolin, Mary Y. Nesbit, and James R. Pearson, provides many suggestions for individual and small group work by children. The chapter, "An instructional Setting for Mathematics Education: A Mathematics Resource Center," by William D. Hedges and Edith McKinnon, emphasizes the physical facilities, materials and equipment, and work space needed to operate a resource center. Instructional procedures and the role of the principal are also considered.

(12) The National Council of Teachers of Mathematics. Enrichment Mathematics for the Grades. Twenty-Seventh Yearbook. Section I: The Elementary School Years. National Council of Teachers of Mathematics. Washington, D.C.: 1963. An invaluable resource guide for developing laboratory materials for motivation, for enriching the basic program, and for going beyond it.

(13) Pethan, R. A. J. The Workshop Approach to Mathematics. New York: St. Martin's Press. 1968. This is a North American revision of a book describing the experiences of the author who is headmaster of a primary school in Bridport, Dorset, England and his staff. The workshop approach, as operated in the regular classrooms of the school, is based on a set of 224 graded cards with materials and apparatus provided for carrying out the activities and experiments. Children work in pairs or in small groups on a number of mathematical topics. Examples of cards representing different topics and levels of ability are reproduced in the book. Sections on the relative amount of time devoted to the textbook and to workshop activities, equipment and costs, and making mathematics apparatus and models are included.

(14) Spitzer, Herbert F. Practical Classroom Procedures for Enriching Arithmetic. St. Louis: Webster Publishing Co. 1956. Contains a wealth of ideas for developing activity cards for individualized and small group learning.

(15) Walter, Marion I. Boxes, Squares and Other Things--A Teachers' Guide for a Unit in Informal Geometry. Washington, D.C.: National Council of Teachers of Mathematics, Inc. 1970. This pamphlet, based chiefly on work with construction paper and milk cartons, offers a wealth of ideas from which teachers could build units for individualized and small group work in a mathematics laboratory.

(16) Yeomans, Edward. Education for Initiative and Responsibility. Boston: National Association of Independent Schools. 1969. A report by the author of his visit to the schools of Liechestershire, England and his reactions to the experimental program in operation which is mainly a laboratory approach emphasizing individual and small group learning by students.

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_____:Shape and Size III. New York: John Wiley & Sons, Inc., 1969.

_____:Pictorial Representation. New York: John Wiley & Sons, Inc., 1967.

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(The aim of this project is to devise a contemporary approach to mathematics for children from 5 to 13.)

School Council for the Curriculum and Examinations. Mathematics in Primary Schools, Curriculum Bulletin, No. 1. London: Her Majesty's Stationery Office, 1966.